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INFERENCE ON HIV AFFECTED PEOPLE THROUGH SHOCK MODEL APPROACH

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ABSTRACT

Since the beginning of the epidemic disease, mathematicians and statisticians have developed models to describe and predict the course of the infection, both at the microbiological level and in a reference population, made up of one or more risk. Here in this paper a three parameter Exponentiated Weibull distribution is been fitted in the shock model approach. The findings of the threshold level with numerical illustration is been given.

Keyword: HIV, Shock Model, Threshold, Three parameter Exponentiated Weibull distribution.

INTRODUCTION

Any component exposed to shocks which cause damage to the component is likely to fail when the total cumulated damage exceed a level called threshold. It may happen that successive shocks become increasingly effective in causing damage, even though they are independent. One is interested in an item for which there is a significant individual variation in ability to withstand shocks. When the immune system is affected in human body, shock with different infected variable is the one to look. When the immune system does not accumulate the increase in shock which is the inter-arrival time, the expected life time of the human system will reach the threshold. Many models with different distribution using shock model approach is derived, one can see more details in Pandiyan *et al.*,(2014), Pandiyan *et al.*,(2013), Subramanian, *etal.*, (2012) and Esary *et al.*,(1973).

A person is exposed to HIV infection. At every epoch of contact with an infected there is some contribution to the antigenic diversity. Anti Retroviral Therapy is administed to the infected. There is a particular level of antigenic diversity of the invading, and it is called the antigenic diversity threshold. If antigenic diversity crosses this threshold the seroconversion takes place. The interarrival times between the successive contacts are random variables which are identically independently distributed.

MODEL DEVELOPED

The survival function $\overline{H}\,(x) = 1 - F(x)$, which is derived from the three parameter Exponential Weibull distribution is given in equation (1), on simplification

$$\overline{H}\,(x) = e^{-\left(\frac{x}{\sigma_1}\right)^{\beta_1}} + e^{-\left(\frac{x}{\sigma_2}\right)^{\beta_2}} - e^{-\left(\frac{x}{\sigma_1}\right)^{\beta_1}} e^{-\left(\frac{x}{\sigma_2}\right)^{\beta_2}} \qquad (1)$$

The shock survival probability is given by

$$P(X_i < Y) = \int_0^\infty g^*(x)\ \overline{H}(x)\,dx$$

$$= g^*\left[\left(\frac{x}{\sigma_1}\right)^{\beta_1}\right]^k + g^*\left[\left(\frac{x}{\sigma_2}\right)^{\beta_2}\right]^k - g^*\left\{\left(\frac{x}{\sigma_1}\right)^{\beta_1} + \left(\frac{x}{\sigma_2}\right)^{\beta_2}\right\}^k \ \textit{by convolation theorem}$$

It may happen that successive shock become increasingly effective in causing damage, even though they are independent. This means that $V_k(t)$, the distribution function of the k^{th} damage is decreasing in k= 1, 2,… for each t. Counting renewal process is been used in equation (2)

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t)\, P(X_i < Y) \tag{2}$$

The life time is given by $L(t) = 1 - S(t)$

Taking Laplace Transformation of the life time L(t), we get equation (3)

$$l^*(s) = \frac{\left[1 - g^*\left(\frac{x}{\sigma_1}\right)^{\beta_1}\right] f^*(s)}{\left[1 - g^*\left(\frac{x}{\sigma_1}\right)^{\beta_1} f^*(s)\right]} + \frac{\left[1 - g^*\left(\frac{x}{\sigma_2}\right)^{\beta_2}\right] f^*(s)}{\left[1 - g^*\left(\frac{x}{\sigma_2}\right)^{\beta_2} f^*(s)\right]} - \frac{\left[1 - g^*\left(\left(\frac{x}{\sigma_1}\right)^{\beta_1} + \left(\frac{x}{\sigma_2}\right)^{\beta_2}\right)\right] f^*(s)}{\left[1 - g^*\left(\left(\frac{x}{\sigma_1}\right)^{\beta_1} + \left(\frac{x}{\sigma_2}\right)^{\beta_2}\right) f^*(s)\right]} \tag{3}$$

Now, $f^*(s) = \frac{\eta}{\eta+s}$ substituting in the above equation (3), on simplification, we get equation (4)

$$= \frac{\eta\left[1 - g^*\left(\frac{x}{\sigma_1}\right)^{\beta_1}\right]}{\left[\eta + s - g^*\left(\frac{x}{\sigma_1}\right)^{\beta_1}\eta\right]} + \frac{\eta\left[1 - g^*\left(\frac{x}{\sigma_2}\right)^{\beta_2}\right]}{\left[\eta + s - g^*\left(\frac{x}{\sigma_2}\right)^{\beta_2}\eta\right]} - \frac{\eta\left[1 - g^*\left(\left(\frac{x}{\sigma_1}\right)^{\beta_1} + \left(\frac{x}{\sigma_2}\right)^{\beta_2}\right)\right]}{\left[\eta + s - g^*\left(\left(\frac{x}{\sigma_1}\right)^{\beta_1} + \left(\frac{x}{\sigma_2}\right)^{\beta_2}\right)\eta\right]} \tag{4}$$

The expected time and variance are hence derived

$$E(T) = \frac{d}{ds} L^*(s) \quad given\ s = 0$$

$$E(T) = \frac{1}{\eta\left[1 - g^*\left(\frac{x}{\sigma_1}\right)^{\beta_1}\right]} + \frac{1}{\eta\left[1 - g^*\left(\frac{x}{\sigma_2}\right)^{\beta_2}\right]} - \frac{1}{\eta\left[1 - g^*\left(\left(\frac{x}{\sigma_1}\right)^{\beta_1} + \left(\frac{x}{\sigma_2}\right)^{\beta_2}\right)\right]}$$

$$g^*(.) \sim \exp(\mu), \qquad g^*\left(\frac{x}{\sigma_1}\right)^{\beta_1} \sim \exp\left(\frac{\mu_1}{\mu_1 + \left(\frac{x}{\sigma_1}\right)^{\beta_1}}\right), \quad g^*\left(\frac{x}{\sigma_2}\right)^{\beta_2} \sim \exp\left(\frac{\mu_2}{\mu_2 + \left(\frac{x}{\sigma_2}\right)^{\beta_2}}\right)$$

$$g^*\left[\left(\frac{x}{\sigma_1}\right)^{\beta_1} + \left(\frac{x}{\sigma_2}\right)^{\beta_2}\right] \sim \exp\left(\frac{\mu_{1+}\mu_2}{\mu_{1+}\mu_2 + \left[\left(\frac{x}{\sigma_1}\right)^{\beta_1} + \left(\frac{x}{\sigma_2}\right)^{\beta_2}\right]}\right)$$

On simplification we get,

$$E(T)=\frac{\sigma_1{}^{\beta_1}\mu_1+x^{\beta_1}}{\eta\, x^{\beta_1}}+\frac{\sigma_2{}^{\beta_2}\mu_2+x^{\beta_2}}{\eta\, x^{\beta_2}}-\frac{\left[\mu_1\sigma_1{}^{\beta_1}\mu_2\sigma_2{}^{\beta_2}+\mu_1\sigma_1{}^{\beta_1}x^{\beta_2}+\mu_2\sigma_2{}^{\beta_2}x^{\beta_1}+x^{\beta_{1+}\beta_2}\right]}{\eta\left[\mu_1\sigma_1{}^{\beta_1}x^{\beta_2}+x^{\beta_{1+}\beta_2}-\mu_1\sigma_1{}^{\beta_1}x^{\beta_2}-\mu_1\mu_2\sigma_2{}^{\beta_2}\sigma_1{}^{\beta_1}\right]} \quad (5)$$

$$V(T)=E(T^2)-[E(T)]^2$$

$$\begin{aligned}V(T)=&\frac{\left[\sigma_1{}^{\beta_1}\mu_1+x^{\beta_1}\right]^2}{\eta^2\left[x^{\beta_1}\right]^2}+\frac{\left[\sigma_2{}^{\beta_2}\mu_2+x^{\beta_2}\right]^2}{\eta^2\left[x^{\beta_2}\right]^2}\\&-\frac{\left[\mu_1\sigma_1{}^{\beta_1}\mu_2\sigma_2{}^{\beta_2}+\mu_1\sigma_1{}^{\beta_1}x^{\beta_2}+\mu_2\sigma_2{}^{\beta_2}x^{\beta_1}+x^{\beta_{1+}\beta_2}\right]^2}{\eta^2\left[\mu_1\sigma_1{}^{\beta_1}x^{\beta_2}+x^{\beta_{1+}\beta_2}-\mu_1\sigma_1{}^{\beta_1}x^{\beta_2}-\mu_1\mu_2\sigma_2{}^{\beta_2}\sigma_1{}^{\beta_1}\right]^2}\\&-\left[\frac{\sigma_1{}^{\beta_1}\mu_1+x^{\beta_1}}{\eta\, x^{\beta_1}}+\frac{\sigma_2{}^{\beta_2}\mu_2+x^{\beta_2}}{\eta\, x^{\beta_2}}\right.\\&\left.-\frac{\left[\mu_1\sigma_1{}^{\beta_1}\mu_2\sigma_2{}^{\beta_2}+\mu_1\sigma_1{}^{\beta_1}x^{\beta_2}+\mu_2\sigma_2{}^{\beta_2}x^{\beta_1}+x^{\beta_{1+}\beta_2}\right]}{\eta\left[\mu_1\sigma_1{}^{\beta_1}x^{\beta_2}+x^{\beta_{1+}\beta_2}-\mu_1\sigma_1{}^{\beta_1}x^{\beta_2}-\mu_1\mu_2\sigma_2{}^{\beta_2}\sigma_1{}^{\beta_1}\right]}\right]^2\end{aligned} \quad (6)$$

CONCLUSION

The mathematical models have been discussed by various authors taking into consideration, many hypothetical assumptions; such models provide the possible clues relating to the consequences of infections, the time taken for seroconversion etc., but these models serve as useful suggestion for the medical personnel to suitably develop the drugs and medicine, and also the methods of treatment.

When increasing the different parameters $\sigma_1, \sigma_2, \mu_1, \mu_2, \beta_1, \beta_2$ along with inter arrival time η, we observe the threshold level to fall down which is seen in the figure 1a, 1b to 6a, 6b. Both the Expected time and variance is observed to be decreasing in all the format of different parameters.

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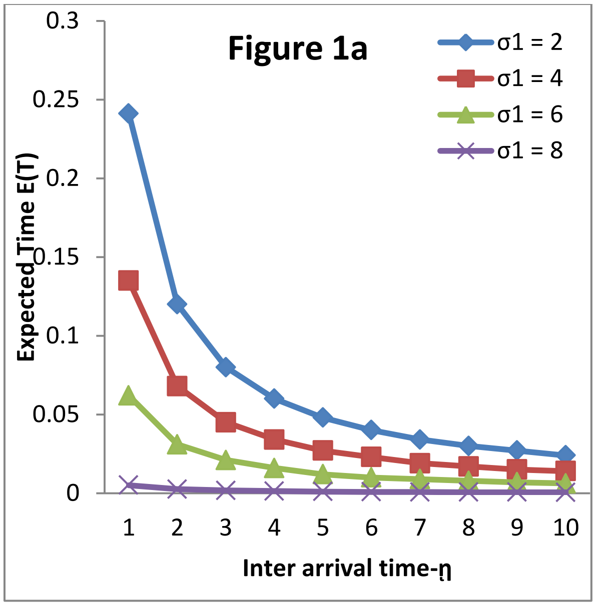

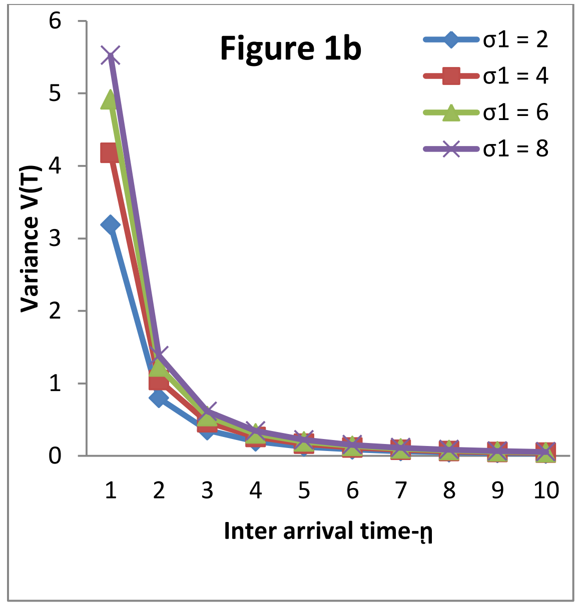

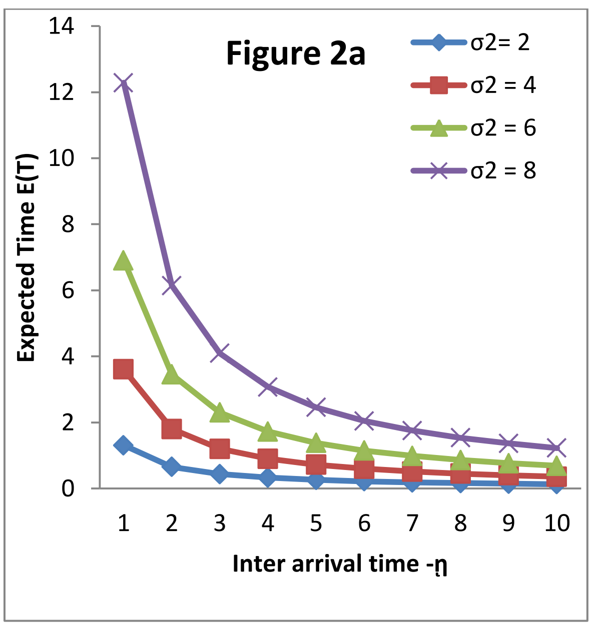

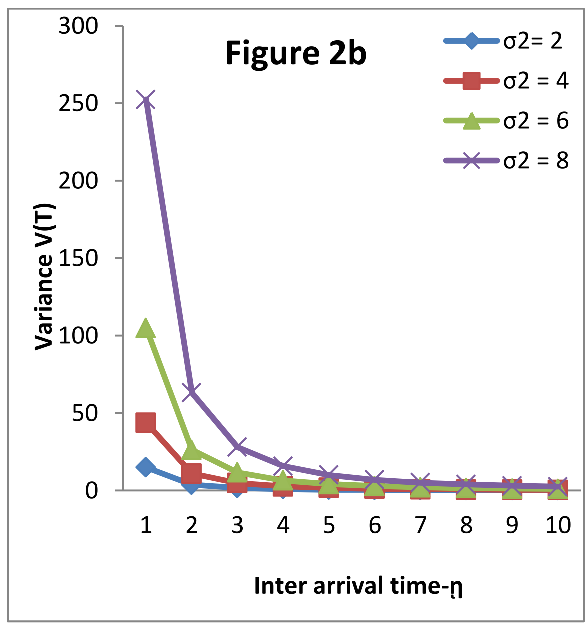

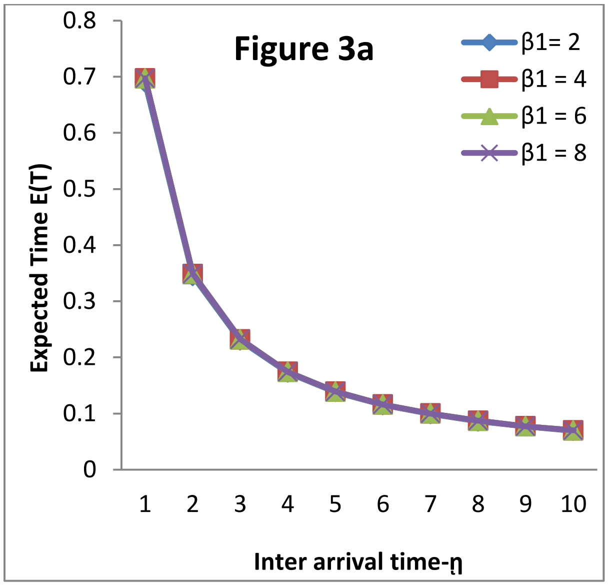
Figure 3a
β1= 2
β1 = 4
β1 = 6
β1 = 8
Expected Time E(T)
Inter arrival time-η

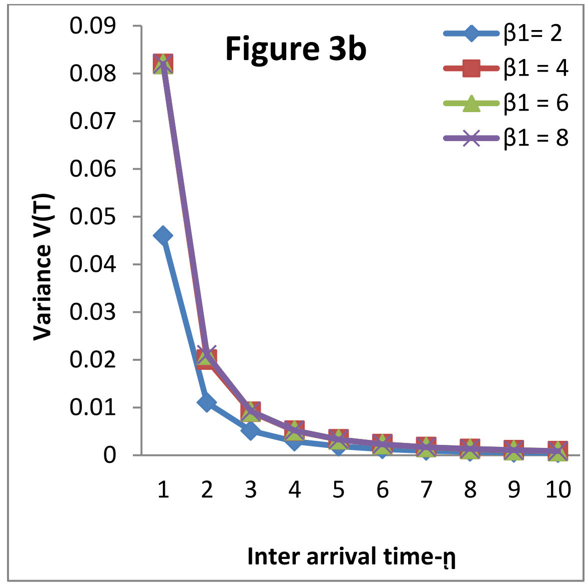
Figure 3b
β1= 2
β1 = 4
β1 = 6
β1 = 8
Variance V(T)
Inter arrival time-η

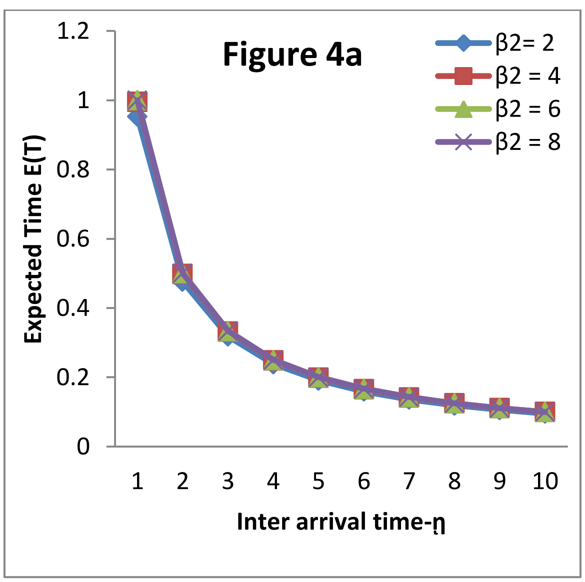
Figure 4a
β2= 2
β2 = 4
β2 = 6
β2 = 8
Expected Time E(T)
Inter arrival time-η

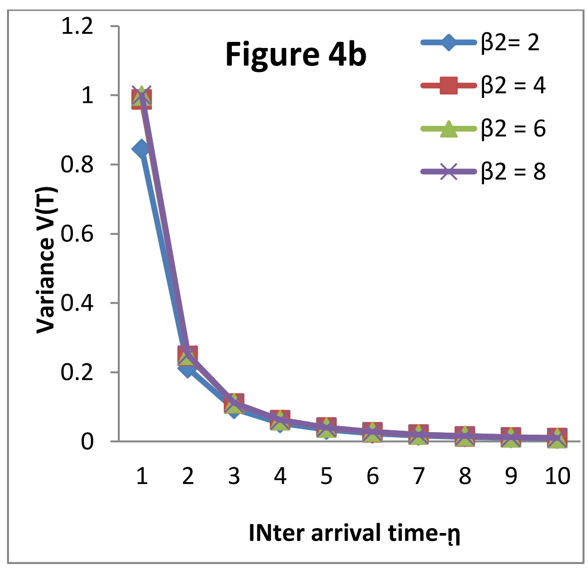
Figure 4b
β2= 2
β2 = 4
β2 = 6
β2 = 8
Variance V(T)
INter arrival time-η

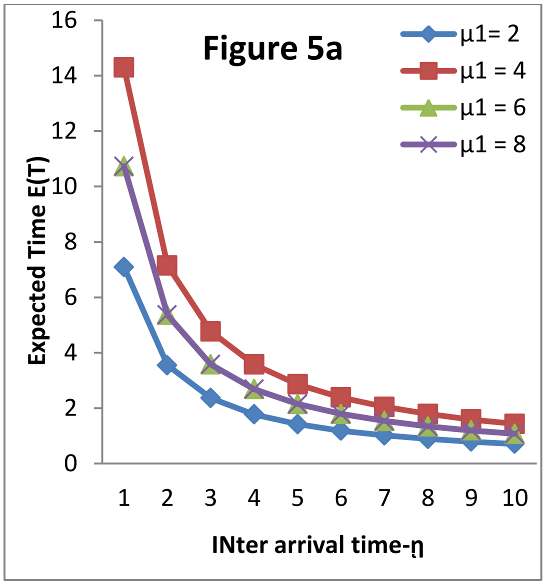
Figure 5a
Expected Time E(T)
INter arrival time-η
μ1= 2
μ1 = 4
μ1 = 6
μ1 = 8

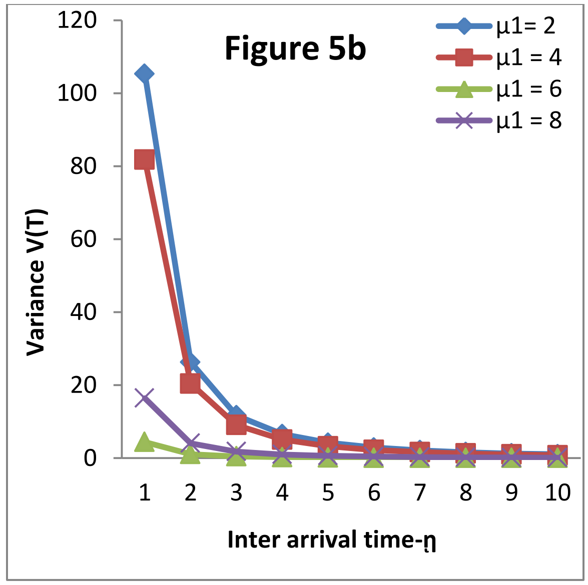
Figure 5b
Variance V(T)
Inter arrival time-η
μ1= 2
μ1 = 4
μ1 = 6
μ1 = 8

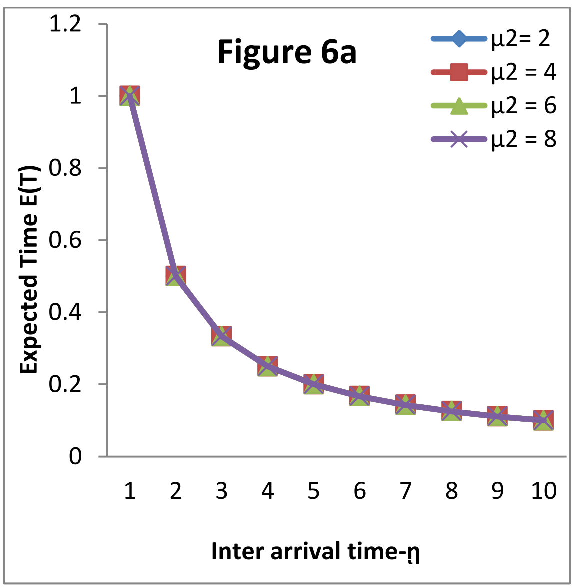
Figure 6a
Expected Time E(T)
Inter arrival time-η
μ2= 2
μ2 = 4
μ2 = 6
μ2 = 8

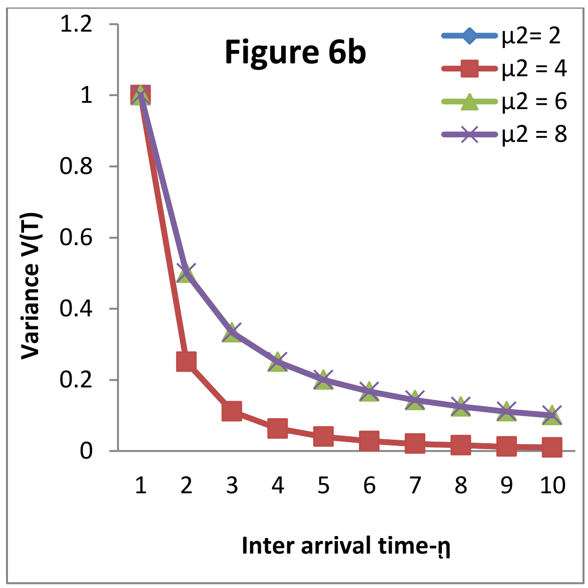
Figure 6b
Variance V(T)
Inter arrival time-η
μ2= 2
μ2 = 4
μ2 = 6
μ2 = 8

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